# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – STATISTICS

THIRD SEMESTER – APRIL 2010

**ST 3811 / 3808 - MULTIVARIATE ANALYSIS**

Date & Time: 21/04/2010 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

## PART – A

**Answer all the questions.** **(10 x 2 = 20)**

1.If X = ( $X_1$, $X_2$ )'~$N_2\left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ then write the c.f. of the marginal distribution of $X_2$.

2. Suggest unbiased estimators for μ and Σ when the sample is from N (μ , Σ ).

3. If $X' = (X_1, X_2) \sim N_2\left\{ \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ then obtain the density of the marginal distribution of $X_1$

4. Define Hotelling's $T^2$ – statistics.

5. Define Fisher's Z-transformation

6. What is factor analysis?

7. Explain the concept of outliers in multivariate data analysis.

8. Outline single linkage procedure.

9. Distinguish between principal component analysis and factor analysis.

10. Explain Q-Q plots.

## PART B

**Answer any FIVE questions.** **(5 x 8 = 40)**

11.Find the multiple correlation coefficient between $X_1$ and $X_2$, $X_3$, … , $X_p$.
Prove that the conditional variance of $X_1$ given the rest of the variables cannot be greater than unconditional variance of $X_1$.

12. Derive the c.f. of multivariate normal distribution.

13. Let Y ~ $N_p$ ( **0** , Σ ). Show that $Y'\Sigma^{-1}Y$ has $\chi 2$ distribution.

14. Obtain a linear function to allocate an object of unknown origin to one of the two normal populations.

15. Let X ~ $N_p$ (μ, Σ). If $X^{(1)}$ and $X^{(2)}$ are two subvectors of X, obtain the conditional distribution of $X^{(1)}$ given $X^{(2)}$.

16.Giving suitable examples explain how factor scores are used in data analysis.

17.Let $(x_i, y_i)'$ , i = 1, 2, 3 be independently distributed each according to bivariate normal with mean vector and covariance matrix as given below. Find the joint distribution of six variables. Also find the joint distribution of $\bar{x}$ and $\bar{y}$ .

Mean vector: (μ, τ)', covariance matrix: $\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$

18. Explain the principal component (principal factor) method of estimating L in the factor analysis method.

## PART C

**Answer any TWO questions.** **(2 x 20 = 40)**

19. a) If $X \sim N_p(\mu, \Sigma)$ then prove that $Z = DX \sim N_p(D\mu, D\Sigma D')$ where D is qxp matrix of rank $q \leq p$.

b). Derive the procedure to test the equality of mean vectors of two p-variate normal populations when the dispersion matrices are equal. ( 10+10)

20.a) What are principal components?. Outline the procedure to extract principal components from a given correlation matrix.

b) What is the difference between classification problem into two classes and testing problem?. (14 + 6)

21.a) Explain in detail $T^2$-Statistic with an illustration.

b) Distinguish between classification and discrimination with an illustration. (12+8)

22.a) Giving a suitable example describe how objects are grouped by complete linkage method.

b) Discuss the effect of an orthogonal transformation in factor analysis method in detail. ( 8 +12 )

************